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Studies on building up strategic management incentive system: incentive mechanism design and structure for monopoly fixed-point hospital in the commercial insurance for medical care

1. Introduction. In 1994 the cities of Jiu-jiang and Zhen-jinag (China) for the first time tried to reform the system of the insurance for medical care. In the success experience foundation, the nationwide reform of the system of the insurance for medical care started officially at the end of 1999, and «the State-Council Decision on Establishing the Basic System of the Insurance of Cities Staff ‘s Medical Care» appeared at the end of the same year. The reform basic mentality is «the low level, the broad cover, both sides undertaking, series account union». The reform has yielded some remarkable results, the preliminary policy system of the insurance for medical care has been established, the unified administration system and the business management system have started to form, and the multi-level system of the insurance for medical care has been consummated gradually. However, with the comprehensive implementation of the policy of the insurance for medical care, the hospital benefit has had an enormous loss. For example, before the medical reform, from 1990 to 1998, the rate of the average outpatient services in the sanitation system used to be 25%, and the rate of hospitalization expenses had increased annually by 23%. After this reform, two rates fell respectively to 9% and 5% in 2001. Therefore, facing the reform of the medical system, the hospital’s attitude is negative, which is a fact that cannot be neglected [2. 6–7, 10, 13].

A hospital provides medical services for the insured, so this is the premise and the key to the operation of the insurance for medical care, and also «the main item» of reducing the medical expense in the process of controlling the insurance risk. Entrusted by the insurance company, the fixed-point hospital must fulfill many duties, such as evaluating the health state of the insured, providing their regular physical examination and supplying the insured patients with medical services. In the process of providing medical services, the hospital and doctor have the initiative; while the patient does not and can only passively accept medical services without bargaining with the hospital, due to a lack of the knowledge and ability to judge the medical services’ quantity and quality beforehand. Meanwhile, the insurance company has also difficulties in managing directly and restricting the medical service provider. This kind of asymmetric information between the insurance company and the fixed-point medical institution inevitably causes excessive consumption of medical resources characterized mainly by the hospital inducing demand, and the constant appearance of the moral hazard such as «making the big prescription». The most typical example is the event of the medical fees as high as 5 million yuans that occurred in the Second Affiliated Hospital of Harbin Medical University in 2005 [17]. Thus, the business of the insurance for medical care has been developing hard, which becomes a global problem [1–2, 6–16].

Nowadays, «the system of controlling fixed-point hospital qualifications» implemented by the domestic insurance company is a decision to renew their next year contracts according to medical expenses and the quality of medical services of the fixed-point hospital. But this loose system makes it difficult to urge the hospital to control diligently medical expenses. With the game theory and the principal-agent theory, the reference [1], [2] and [3] discuss the feasibility and stability of cooperation between the insurance company and the hospital in the risk-control aspect, and advise to establish an incentive mechanism in which «the risk is altogether taken on, profits are shared» between the insurance company and the fixed-point hospital. However, the concrete form of this incentive mechanism is not given in these references. The reference [4] and [5] have made a preliminary analysis of this issue and proposed a tentative plan: when the insurance company has a surplus in the business of the insurance for medical care, the majority of profits should be used in rewarding the fixed-point hospital for observing the contract; when the insurance company has a loss, a part of this loss should be borne by the fixed-point hospital for breaking the contract. This article will thoroughly analyze this question using the analytic mentality of the principal-agent theory and the mathematical method.

2. Model fundamental assumption. At present, the traditional principal-agent theory is the most effective incentive theory. The prime target of this theory is to help the principal to seek an optimum incentive mechanism, in order to share the risk between the principal and the agent, and to prevent or reduce the occurrence of the agent's moral hazard and reverse choices. Undoubtedly, there is a principal-agent relationship between the insurance company and the fixed-point hospital. Moreover, there is a lot of moral-risk behavior. For example, the fixed-point hospital induces the high-risk group to participate in the insurance for medical care through reducing the physical examination standard for the insured. The doctor writes «the big prescription», and tricks the insurance company as well as the patient [6–7, 10–17]. But, this kind of the principal-agent relationship is different from the general principal-agent relationship. As a partnership between the insurance company and the fixed-point hospital is neither a relationship between a superior and subordinate in the process of controlling the capital, nor a close direct economical relationship, it is difficult to form the principal-agent partnership in which «the risk is altogether undertaken, profits are shared» between them. Thus, the insurance company lacks the effective method of restraining the hospital [1].

However, the analytical mentality and method of the principal-agent theory may provide certain theoretical references for the insurance company to restrain the fixed-point hospital and control the insurance risk. With the help of this analytical mentality and method the article will study how the insurance company stimulates the fixed-point hospital in the monopolistic market of the medical services. Beginning the study, it is necessary to make the following hypotheses.

Hypothesis 1: Given the scarcity of medical resources in a small county city and the countryside, it is assumed that the medical service market is a monopoly, and the fixed-point hospital is the only one in the area of its responsibility. Thus, the insured patients can get medical services in the local fixed-point hospital only.

Hypothesis 2: Taking the government's participation and some compulsory policies into consideration, it is presumed that the cooperation between the insurance company and the fixed-point hospital is long-term and is not to be broken, which is suitable for the cooperative insurance for medical care in the socialized medical insurance at least.

Hypothesis 3: Since in the business of the insurance for medical care the fixed-point hospital is only the third party and also earns its own income during a cooperation with the insurance company, it is assumed that the negative utility of the fixed-point hospital's diligence (also called «the diligent cost») is approximately equal to zero in the process of controlling the insurance risk.

Hypothesis 4: Since the level of fixed-point hospital's efforts is supervised by the insurance company acting as a principal in the process of controlling the insurance risk with difficulty, this level may be indirectly inferred through the insurance company's own profits and the loss ratio. Thus, the proportion in which the fixed-point hospital shares the insurance profits may also be determined by the insurance company's own profits and the loss ratio. Moreover, it is presumed that this proportion is $Q(r)$ $(0 \le Q(r) \le 1)$, of which the loss ratio $r\,(0 \le r \le 1)$ is the proportion of the total amount of the medical compensation money to the total premium of the insurance for medical care in the region of the fixed-point hospital's responsibility. The size of the loss ratio, on the one hand, relates to the level of subjective efforts of the fixed-point hospital; on the other hand, refers to the insured own physical condition and his (or her) hypothetical quality, etc. Under the same exterior factors, the higher the level of fixed-point hospital's efforts $a\,(a \geq 0)$ is the lower the loss ratio r is, i.e. $\frac{\partial r}{\partial a} \leq 0$. But with the unceasing increasing of the level of the fixed-point hospital's efforts, the degree of decreasing of the loss ratio will slow down, i.e. $\frac{\partial^2 r}{\partial a^2} \geq 0$. Meanwhile, we supposes $Q'(r) \leq 0$ and $Q''(r) \geq 0$. Therefore, $\frac{\mathrm{d}Q(r)}{\mathrm{d}a} = \frac{\mathrm{d}Q(r)}{\mathrm{d}r}\frac{\partial r}{\partial a} \geq 0$, $\frac{\mathrm{d}^2 Q(r)}{\mathrm{d}a^2} = \frac{\mathrm{d}^2 Q(r)}{\mathrm{d}r^2}\left(\frac{\partial r}{\partial a}\right)^2 + \frac{\mathrm{d}Q(r)}{\mathrm{d}r}\frac{\partial^2 r}{\partial a^2} \leq 0$, which means that the proportion $Q(r(a))$ is an increasing and marginal decreasing function of the level of fixed-point hospital's efforts a. In addition, given that the proportion $Q(r)$ should be simple and feasible, $Q(r)$ must be a simpler elementary function.

3. Model establishment and analysis. It is assumed that the total premium of the insurance for medical care is π, and the loss ratio is r, and the annual fee of the insurance company for management is C in the region of the fixed-point hospital's responsibility. Generally, the management fee is mainly relative to the interior system of the insurance company and compensatory times, but has nothing to do with the loss ratio. If the insurance company has a surplus in the business of the insurance for medical care in the region of the fixed-point hospital's responsibility in the year's end, it can reward the fixed-point hospital for cooperation according to the following contract.

$y = \pi(1-r)Q(r) \quad (0 \leq r \leq 1)$ (1)

The profits of the insurance company in this region are:

$s = \pi - \pi r - y - C$ (2)

With regard to the present domestic main way to compensate the insured for the medical services, as the medical expenses of the insured are divided into the hospitalization fees and the outpatient fees, the former can be reimbursed through the insurance company or the insurance bureau appropriating directly to the hospital account; and only if the insured pays in advance, can the latter be reimbursed through their appropriating to the individual account [5,8–9,12–15]. Under the hypothesis 1, the money of the insurance company for medical compensation will completely transfer the fixed-point hospital by the insured. Thus, the real gross income of the fixed-point hospital is

$w = y + \pi r$ (3)

Proposition 1. There is no real-valued function $Q(r)$ $(0 \leq Q(r) \leq 1)$ that is the decreasing function of s and w, and also the increasing function of a.

Proved: $\because$ $s = \pi - w - C$. $\frac{\mathrm{d}s}{\mathrm{d}r} = -\frac{\mathrm{d}w}{\mathrm{d}r}$; $\frac{\mathrm{d}s}{\mathrm{d}r}\frac{\partial r}{\partial a} = \frac{\mathrm{d}s}{\mathrm{d}a} = -\frac{\mathrm{d}w}{\mathrm{d}a} = -\frac{\mathrm{d}w}{\mathrm{d}r}\frac{\partial r}{\partial a}$

$\therefore$ If $\frac{\mathrm{d}s}{\mathrm{d}r} > 0$, then $\frac{\mathrm{d}w}{\mathrm{d}r} < 0$, $\frac{\mathrm{d}s}{\mathrm{d}a} \leq 0$, $\frac{\mathrm{d}w}{\mathrm{d}a} \geq 0$; if $\frac{\mathrm{d}s}{\mathrm{d}r} < 0$, then $\frac{\mathrm{d}w}{\mathrm{d}r} > 0$, $\frac{\mathrm{d}s}{\mathrm{d}a} \geq 0$, $\frac{\mathrm{d}w}{\mathrm{d}a} \leq 0$; if $\frac{\mathrm{d}s}{\mathrm{d}r} = 0$, then $\frac{\mathrm{d}w}{\mathrm{d}r} = 0$, $-1 + Q(r) - (1-r)Q'(r) = 0$, solve this differential equation, $Q(r) = k(1-r)^{-1} + 1$, $k\,(k > 0)$ is an integration constant. Obviously, $Q(r) > 1$, which is contradictory with the condition that $Q(r)$ meets. □

The following conclusion may be drawn by the proposition 1.

Conclusion 1. In the monopolistic market of medical services, there is no optimum incentive mechanism to enable both the insurance company and the fixed-point hospital to obtain profits.

In other words, both proposition 1 and conclusion 1 explain together that under hypothesis 1 there is no optimum incentive contract, which not only enables the fixed-point hospital to control diligently the insurance risk on its own initiative to reduce the loss ratio, but also enables the insurance company to obtain more profits at a lower loss ratio. The non-existence of this optimum incentive contract roots in the fact that the total compensatory money of the insurance company is transferred directly or indirectly to the fixed-point hospital by the insured patients.

However, there is the second-best incentive mechanism. In fact, the prime purpose of introducing the loss ratio in the cooperation contract is to make the fixed-point hospital control positively the insurance risk under the motive of raising its own income, and avoid or reduce diligently the occurrence of the moral hazard such as «writing the big prescription», making non-essential physical examinations, requesting the insured patients to be re-hospitalized, etc. Therefore, that is OK so long as a real-valued elementary function $Q(r)$ that makes w becomes a function of r at the certain interval sought.

Proposition 2. (1) If $Q(r) \equiv K$ (constant, $0 \le K \le 1$), w will be a monotone increasing function at the interval of $[0,1]$; s will be a monotone decreasing function at the interval of $[0,1]$.

(2) If $Q(r)$ satisfies the following condition at the interval of $[0,1]$, $0 \le Q(r) \le 1$, $Q'(r) < 0$, $Q''(r) \ge 0$, $1 - Q(0) + Q'(0) < 0$, there will be unique $\xi \in (0,1)$, *s.t.* if $r \in [0,\xi]$, w will become a strict monotone decreasing function, s will become a strict monotone increasing function; if $r \in [\xi,1]$, w will become a strict monotone increasing function, s will be come a strict monotone decreasing function.

Proved: (1) If $Q(r) \equiv K$, then $w = \pi(1-r)K + \pi r$, $w' = \pi(1-K) \ge 0$, w will be a monotone increasing function at the interval of $[0,1]$. According to $s'(r) = -w'(r)$, s will be a monotone decreasing function at the interval of $[0,1]$.

(2) $\because$ $w = \pi(1-r)Q(r) + \pi r$, $w' = \pi[1 - Q(r) + (1-r)Q'(r)]$, $w'' = \pi[(1-r)Q''(r) - 2Q'(r)] > 0$

$\therefore$ $w'(r)$ is a strict monotone increasing function at the interval of $[0,1]$, and $w'(0) \le w'(r) \le w'(1)$.

$\because$ $w'(0) = \pi[1 - Q(0) + Q'(0)] < 0$, $w'(1) = \pi[1 - Q(1)] > 0$. ($Q(1) \ne 1$, otherwise $1 = Q(1) \le Q(r) \le 1$ by $0 \le Q(r) \le 1$ and $Q'(r) < 0$, i.e. $\forall r \in [0,1]$, $Q(r) = 1$. That is contradictory.

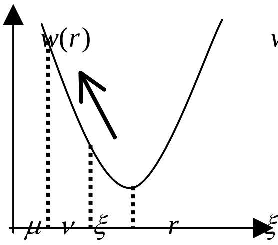
According to the zero-point existence theorem of the continuous function and the strict monotony of $w'(r)$, there will be unique $\xi \in (0,1)$, *s.t.* $w'(\xi) = 0$. Therefore, if $r \in [0,\xi]$, then $w'(r) < 0$, w will be a strict monotone decreasing function, s will be a strict monotone increasing function; if $r \in [\xi,1]$, then $w'(r) > 0$, w will be a strict monotone increasing function, s will be a strict monotone decreasing function. □

The following conclusion can be made by the proposition 2.

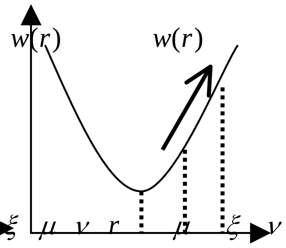
Conclusion 2. In the monopolistic market of the medical services, using the fixed incentive coefficient (also called the proportion of drawing bonuses) stimulates the fixed-point hospital to control diligently the insurance risk and avoid or reduce the occurrence of the moral hazard with difficulty; to some extent, using the floating incentive coefficient urges probably the fixed-point hospital to control the insurance risk and avoid or reduce the occurrence of the moral hazard on its own initiative. But, the insurance company has to make concessions in the question of its own income maximization in order to encourage the fixed-point hospital to control positively the insurance risk.

The proposition 2 together with the proposition 1 reveals that there is some contradiction between the insurance company and the fixed-point hospital during the game of the risk control. Since the insurance company has to cooperate with the fixed-point hospital under the hypothesis 2, the equilibrium solution of the game exists inevitably. Whether this equilibrium solution is optimum has direct relation with the random distribution state of the loss ratio.

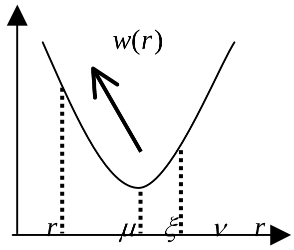
Owing to the uncertainty of the insured getting sick and the uncertainty of the medical result, the loss ratio has randomness. The random distribution state of this ratio is influenced by the degree of the fixed-point hospital's efforts. Moreover, it is assumed that the expectation of this random variable is a function of the level of the fixed-point hospital's efforts, i.e. $E(r)=\theta(a)$. By $\frac{\partial r}{\partial a}\le 0$, then $\frac{\partial \theta}{\partial a}\le 0$, namely, the higher the degree of fixed-point hospital's efforts is, the lower the average loss ratio is. In addition, it is presumed that $0\le\mu=\inf_a \theta(a)\le\theta(a)\le\sup_a \theta(a)=\nu<1$. Subsequently, we will analyze how the relation between the minimum point ξ and the infimum μ and the supremum ν affects this special principal-agent relation by the graphical method.



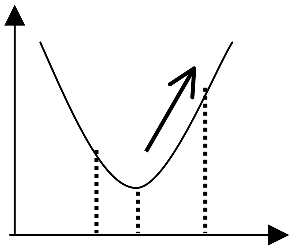
(Chart 1)



(Chart 2)



(Chart 3)



(Chart 4)

According to the chart 1, if ξ falls on the right side of ν, the fixed-point hospital will tend to control positively the insurance risk in order to maximize its own income. According to the chart 2, if ξ falls on the left side of μ, the fixed-point hospital will not have any activeness and initiative in the process of controlling the insurance risk in order to obtain more income. According to the chart 3, if ξ falls between μ and ν, and approaches ν, the fixed-point hospital may obtain more income through strengthening the control of the insurance risk than through relaxing the control of the insurance risk. In this situation, the fixed-point hospital will also have a tendency to cooperate with the insurance company on its own initiative. According to the chart 4, if ξ falls between μ and ν, and approaches μ, on the contrary, the fixed-point hospital may obtain less income through strengthening the control of the insurance risk than through relaxing the control of the insurance risk. In this situation, the fixed-point hospital will not have any enthusiasm for the control of the insurance risk either. In summary, only if ξ approaches ν and is far away from μ, it is probable that the fixed-point hospital controls diligently the insurance risk.

In accordance with the above analysis, how the insurance company stimulates the monopoly fixed-point hospital, the problem can transfer the following optimization problem.

$$\max_{Q(r)\in\Box(r)} \pi(1-r)-\pi(1-r)Q(r)-C$$
$$s.t.\quad \max_a \pi(1-r)Q(r)+\pi r \qquad (4)$$
$$\pi(1-r)-\pi(1-r)Q(r)-C\ge\overline{S}$$

Among this model, the first restricted condition is the incentive compatibility constraint for the fixed-point hospital. This condition explains that once the insurance company gives the proportion of drawing bonus, the fixed-point hospital will consider whether it should make efforts or not, and what level of efforts enables it to obtain more income. In other words, when $Q(r)$ is given, the fixed-point hospital will choose a so that $\frac{\mathrm{d}w}{\mathrm{d}a}=0$. Because of $\frac{\mathrm{d}w}{\mathrm{d}a}=\frac{\mathrm{d}w}{\mathrm{d}r}\frac{\partial \mathrm{r}}{\partial a}$, $\frac{\mathrm{d}w}{\mathrm{d}a}=0$ is equivalent to $\frac{\mathrm{d}w}{\mathrm{d}r}=0$ when $\frac{\partial \mathrm{r}}{\partial a}<0$, i.e. $\exists\xi\in\{r;0<r<1,\ |r-\upsilon|\le|r-\mu|\}$, $s.t.\ 1-Q(\xi)+(1-\xi)Q'(\xi)=0$. The second restricted condition is the constraint of the lowest income control for the insurance company, of which $\overline{S}$ is the lowest income control line of the insurance company. By the conclusion 2, the lowest line cannot excessively be high, otherwise, the second-best contract will not exist, and this principal-agent model will not be solved. Under the hypothesis 2, the fixed-point hospital must cooperate with the insurance company, so the above model has no participation constraint for the agent.

Proposition 3. $\forall \xi \in (0,1)$, $\exists Q(r)$ which is a real-valued elementary function, *s.t.* $0 \le Q(r) \le 1$, $Q'(r) < 0$, $Q''(r) \ge 0$ and $1 - Q(\xi) + (1-\xi)Q'(\xi) = 0$.

Proved: Construct a function $g(r) = (1-kr)^t (0 \le r \le 1; 0 < k \le 1; t \ge 1)$. $g'(r) = -kt(1-kr)^{t-1} < 0$, $g''(r) = k^2 t(t-1)(1-kr)^{t-2} \ge 0$. Therefore, $g(r)$ is a strict monotone decreasing function at the interval of $[0,1]$, and satisfies $0 < (1-k)^t = g(1) \le g(r) \le g(0) = 1$.

Define a function $G(r) = 1 - g(r) + (1-r)g'(r)$ $(0 \le r \le 1)$. $G'(r) = -2g'(r) + (1-r)g''(r) > 0$, so $G(r)$ is a strict monotone increasing function at the interval of $[0,1]$, and satisfies $G(0) \le G(r) \le G(1)$. Because $G(0) = -kt < 0$ and $G(1) = 1-(1-k)^t > 0$, according to the zero-point existence theorem of the continuous function and the strict monotony of $G(r)$, there will be unique $\xi \in (0,1)$, *s.t.* $G(\xi) = 0$, i.e. $1-(1-k\xi)^t - kt(1-\xi)(1-k\xi)^{t-1} = 0$.

Consider a function $\Psi(r) = 1 - g(r) + (1-kr)g'(r)$ $(0 \le r \le \frac{1}{k})$, and we will discover $\Psi'(r) = -2g'(r) + (1-kr)g''(r) > 0$, and that $\Psi(r)$ is also a strict monotone increasing function at the interval of $[0,1]$ and satisfies $\Psi(0) \le \Psi(r) \le \Psi(1)$. Because $\Psi(0) = -kt < 0$ and $\Psi(\frac{1}{k}) = 1 > 0$, again according to the zero-point existence theorem of the continuous function and the strict monotony of $\Psi(r)$, there will be unique $\zeta \in (0, \frac{1}{k})$, *s.t.* $\Psi(\zeta) = 0$, i.e. $\zeta = \frac{1}{k}\left[1 - \left(\frac{1}{1+kt}\right)^{\frac{1}{t}}\right]$. With the method to fetch the logarithm, $\lim\limits_{t\to\infty} \zeta = \frac{1}{k}\left[1 - \lim\limits_{t\to\infty}\left(\frac{1}{1+kt}\right)^{\frac{1}{t}}\right] = 0$. With L'Hospital rule, $\lim\limits_{k\to 0} \zeta = \lim\limits_{k\to 0} \frac{1 - \left(\frac{1}{1+kt}\right)^{\frac{1}{t}}}{k} = \lim\limits_{k\to 0}\left(\frac{1}{1+kt}\right)^{\frac{1}{t}+1} = 1$.

Consider an absolute error $|\Psi(r) - G(r)| = k(1-k)tr(1-kr)^{t-1}$. According to $\left|(1-kr)^{t-1}\right| \le 1$ and the conclusion that the product of an infinitesimal quantity and a bounded quantity is an infinitesimal quantity, $\lim\limits_{k\to 0}|\Psi(r) - G(r)| = \lim\limits_{k\to 0} k(1-k)tr(1-kr)^{t-1} = rt \lim\limits_{k\to 0} k(1-k) = 0$. With L'Hospital rule again, $\lim\limits_{t\to\infty}|\Psi(r) - G(r)| = k(1-k)r \lim\limits_{t\to\infty} \frac{t}{(1-kr)^{1-t}} = k(1-k)r \lim\limits_{t\to\infty} \frac{1}{-(1-kr)^{1-t}\ln(1-kr)} = 0$. Therefore, $\forall \varepsilon > 0$, $\exists k^* \in (0,1]$, $t^* \in [1,\infty)$, *s.t.* $|\Psi(r) - G(r)| < \varepsilon$, in other words, so long as k and t are suitability chosen, the absolute error of $\psi(r)$ and $G(r)$ at the interval of $[0,1]$ will be as small as possible. Thus ζ may be taken as an approximate zero point of $G(r)$, i.e. $\xi \approx \frac{1}{k}\left[1 - \left(\frac{1}{1+kt}\right)^{\frac{1}{t}}\right]$. Because ζ may change at the interval of $(0,1)$, ξ may also do.

By the above analysis, $\forall \xi \in (0,1)$, $\exists Q(r) = (1-kr)^t$ (of which the undetermined coefficient $k\,(0 < k \le 1)$ and $t\,(t \ge 1)$ are determined by $\xi = \frac{1}{k}\left[1 - \left(\frac{1}{1+kt}\right)^{\frac{1}{t}}\right]$), *s.t.* $Q(r)$ satisfies the following properties at the interval of $[0,1]$: $0 \le Q(r) \le 1$, $Q'(r) < 0$, $Q''(r) \ge 0$, $1 - Q(\xi) + (1-\xi)Q'(\xi) = 0$. □

Proposition 4. $\forall \upsilon\,(0 < \upsilon < 1)$, $\exists Q(r)$ which satisfies the conclusion of the proposition 3, so that $w(r(a))$ is an increasing function of a when $r \in [0, \upsilon]$.

With the conclusion of the proposition 3, the proposition 4 can be proved through fetching $\xi = \upsilon$.

The proposition 3 and the proposition 4 prove indirectly the existence of the theory solution to the optimization problems (4). However, the feasible solution to the optimization problems (4) has to be solved according to the actual need.

By the proposition 4, the contract as the expression (1) represents can be modified into the following layout.

$$y = \begin{cases} \pi(1-r)Q(r) & (if \quad 0 \le r \le \upsilon < 1) \\ \pi(\upsilon - r) & (if \quad r > \upsilon) \end{cases} \qquad (5)$$

In the modified contract, the point of discontinuity $\upsilon = \sup_a \theta(a)$ may be replaced with the statistical mean which is obtained according to the historical data of the loss ratio, or be determined by the agreement between the insurance company and the fixed-point hospital. Of course, $Q(r)$ has the direct relation with the selection of υ. Just as the expression (5) represents, if r doesn't exceed υ, the fixed-point hospital will draw the bonus according to the proportion $Q(r)$; if r exceeds υ, the fixed-point hospital will be fined $\pi(r-\upsilon)$. From the gross income of the fixed-point hospital, in other words, the part of excessive expenditures will voluntarily be undertaken by the fixed-point hospital, which is similar with «the method of the total amount control» [2–14] -- the mainstream settlement way of the insurance for medical care in our country and even the world.

Proposition 5. If the modified contract is used, there will be a function $Q(r)$ that satisfies the proposition 4 and has permanently the following properties: $\forall r \in [0, \nu]$, $\forall R \in (\nu, \infty)$, $w(r) > w(R)$.

Proved: According to the proposition 4, there is such a function $Q(r)$ that $w(r)$ is a strict monotone decreasing function at the interval of $[0, \nu]$ and reaches the minimum value $w(\nu) = \pi(1-\nu)Q(\nu) + \pi\nu$. $\forall R \in (\nu, \infty)$, obviously, $w(R) = \pi\nu \le w(\nu)$. □

The proposition 5 explains that using the bonus-penalty contract as the expression (5) represents, can urge the fixed-point hospital to have the full enthusiasm to control the loss ratio below υ in the region of its own responsible. Again according to the proposition 1, 2, 3, 4 and 5, the following conclusion may be reached.

Conclusion 3. In the monopolistic medical service market, there is a sub-optimal incentive contract that enables the fixed-point hospital to cooperate positively with the insurance company on the insurance risk control.

3. Calculating the example.

Just as the next table 1 shows, through the statistical analysis to the revenue and expenditure situation in the health insurance system of our country in the near three years, the loss ratio of the health insurance of our country fluctuates about 0.33 in recent years.

Table 1. The revenue and expenditure situation of the health insurance of our country in the near three years

The beginning and end month	Total income (ten thousand Yuan)	Total compensation money (ten thousand Yuan)	The loss ratio
2003-01 to2003-06	1281240.02	319514.24	0.25
2003-06 to 2003-12	1138001.25	379504.59	0.33
2004-01 to 2004-06	1363776.41	435973.92	0.32
2004-06 to 2004-12	1234994.27	455057.98	0.37
2005-01 to 2005-06	1598017.39	526341.98	0.33
2005-06 to 2005-12	1525002.01	552817.66	0.36
2005-12 to 2006-01	306403.14	104822.99	0.34

Note: In the above table, the primary data stem from the net of China Insures Regulatory Commission, http://www.circ.gov.cn.

The insurance for medical care, one of health insurances, has higher loss ratio than other life insurance and the property insurance. In the light of it, we might as well fetch $\nu = \sup_a \theta(a) = 0.5$, $\mu = \inf_a \theta(a) = 0.2$ and the following the coefficient of drawing bonus.

$$q(r) = k(1-r) \quad (0 \le r \le 1;\ 0 < k \le 1) \qquad 6$$

Thus, the principal-agent model pattern (4) between the insurance company and the fixed-point hospital may be simplified into the following layout.

$$\max_{k \in [0,1]} \pi(1-r) - \pi k(1-r)^2 - C$$
$$s.t. \quad \max_a \ \pi k(1-r)^2 + \pi r \qquad (7)$$
$$\pi(1-r) - \pi k(1-r)^2 - C \ge \overline{S}$$

Proposition 6 $q(r) = k(1-r)$ $(0 \le r \le 1;\ 0 < k \le 1)$ has the following properties: (1) $0 \le q(r) \le 1$, $q'(r) < 0$, $q''(r) > 0$; (2) the equation $1 - q(r) + (1-r)q'(r) = 0$ has the unique solution in the interval of $[0, 0.5]$, which will ergode any value in the interval of $[0, 0.5]$ when k ergodes any value in the interval of $[0.5,1]$.

According to the proposition 2, 3 and 4, the conclusions of the proposition 6 are quite obvious, so the proof can be omitted.

Because of $\frac{\partial r}{\partial a} < 0$ in the ordinary circumstances, the constraint $\max_a \ \pi k(1-r)^2 + \pi r$ can transfer the following equivalent form: $\forall \xi \in \{r; 0 < r < 1, \text{and } |r-0.5| \le |r-0.2|\}$, $w'(\xi) = 0$, i.e. $\xi = 1 - \frac{1}{2k}$. Therefore, $\xi = 1 - \frac{1}{2k} \ge 0.35$, i.e. $k \ge 0.769$. If $C = 0.001\pi$, $\overline{S} = 0.3\pi$ ($\overline{S}$ can't is excessively high, otherwise, the feasible region is null, and the optimization problems (7) doesn't have solution), the constraint $\pi(1-r) - \pi k(1-r)^2 - C \ge \overline{S}$ will change into $k \le \frac{(1-r) - 0.301}{(1-r)^2}$. Thus, $k \le \inf_{0.2 \le r \le 0.5} \frac{(1-r) - 0.301}{(1-r)^2} = 0.796$, and the expression (7) will change into $\max_{0.769 \le k \le 0.796} \pi(1-r) - \pi k(1-r)^2 - 0.001\pi$. When $k = 0.769$, obviously, the objective function reaches the maximum value, i.e. $\max_{0.769 \le k \le 0.796} \pi(1-r) - \pi k(1-r)^2 - 0.001\pi = \pi(1-r) - 0.769\pi(1-r)^2 - 0.001\pi$. Thus, we can attain the optimal coefficient $q(r) = 0.769(1-r)$ $(0 \le r \le 1)$, and the following second-best incentive contract.

$$y = \begin{cases} 0.769\pi(1-r)^2 & (if \quad 0 \le r \le 0.5) \\ \pi(0.5-r) & (if \quad r > 0.5) \end{cases} \qquad 8$$

4. Conclusions. Until now the problem of the risk control of the insurance for medical care is still too difficult in the whole world. The issue how to prevent the moral hazard of the fixed-point hospital is especially important in the process of controlling the insurance risk. Besides strengthening medical education with medical ethics and reinforcing the surveillance in the process of practicing medicine which can restrain the fixed-point hospital from the occurrence of the moral hazard to some extent, there is also the question whether one kind of the incentive mechanism is probably sought so as to urge the fixed-point hospital to reduce positively the occurrence of the moral hazard in the process of cooperation between the insurance company and the fixed-point hospital. In this regard, the article studies mainly how the insurance company stimulates the fixed-point hospital in the monopolistic medical service market. Under the four reasonable hypotheses, just as the expression (4) shows, the principal-agent model between the insurance company and the fixed-point hospital is established; and the following three conclusions are drawn by the strict mathematical derivation. (1) In the monopolistic market of medical services, there is no optimum incentive mechanism to enable both the insurance company and the fixed-point hospital to obtain profits. (2) In the monopolistic market of the medical services, using the fixed incentive coefficient stimulates the fixed-point hospital to control diligently the insurance risk and avoid or reduce the occurrence of the moral hazard with difficulty; to some extent, using the floating incentive coefficient urges probably the fixed-point hospital to control the insurance risk and avoid or reduce the occurrence of the moral hazard on its own initiative. But, the insurance company has to make concessions in the question of its own income maximization in order to rouse the fixed-point hospital to control positively the insurance risk. (3) In the monopolistic medical service market, there is a sub-optimal incentive contract, as the expression (5) shows, which enables the fixed-point hospital to cooperate positively with the insurance company on the insurance risk control so that medical resources are reasonably used and probably the business of the insurance for medical care is deeply developed. At last, just as the expression (8) shows, a feasible contract is given according to some historical statistical data of our country.

In order to make the model be closer to actualities and the derived sub-optimal contract have more incentive function, it is worth considering the introduction of other observable variables such as the loss ratio of the last year in the region of the fixed-point hospital's responsible, the simultaneous loss ratio in the region of another fixed-point hospital's responsible. Due to the limited space in the article, studies of this expansion model as well as the situation in the competitive market of medical services will be handled in a separate article.

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